Student Choice of Computation Methods

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This study was designed to refine an instrument to explore the relationships between students' mental, written and calculator strategies of computation. A theoretical framework for studying the relationships between these three main methods of computation was postulated. This paper describes this model of the computational processes and some preliminary work carried out with middle to upper primary students in Western Australia. In addition, the questions of factors such as access to calculators, emphasis on written algorithms and mental computation influencing choice of computation method are also discussed.

Introduction

The current focus on numeracy and the 'numeracy benchmarks' indicates that there is still a strong focus on number and in particular mental computation and written algorithms, not only in Australia but also elsewhere. The calculator, while it has been widely available in educational settings for well over two decades, often comes in a poor third to the computation mainstays of mental and written calculation. Surveys of teachers' attitudes (Warren & Ling, 1995; Sparrow & Swan, 1997; Jones & Tanner, 1997) on the use of calculators in the nineties indicated that resistance to calculator use in classrooms is quite pronounced. When calculators are used in primary mathematics it is often for checking work or in other low-level contrived situations. There appears to be a reluctance on the part of teachers to allow easy access to the use of calculators for fear that students might become too reliant on them, or that they might make inappropriate choices as to when and how to use them. The Calculator Aware Number (CAN) project carried out in the UK, however, found the opposite to be true (Shuard, Walsh, Goodwin & Worcester, 1991). Children often prided themselves on being able to perform a calculation mentally rather than resort to the use of pen and paper or calculator. Ruthven (1995) made this observation about children who participated in the CAN project.

> The project children were better able to tackle real-world problems and computational tasks; in particular, . . .while they did not make more use of calculators, they made more appropriate choices of calculating device and were better able to interpret their answers.(p. 239)

This raises questions as to what constitutes an appropriate choice, but we do not intend to debate this issue here, other than to state that appropriate choices will vary from child to child, depending upon aspects such as background and experience. For example, we would suggest that computational choice is skewed in favour of written methods due to the strong emphasis on written computation in primary school and the amount of time spent on teaching formal, written algorithms (Porter, 1989).

At first it may appear as though the choice of computational approach is relatively simple and clear-cut but, in reality, many complex factors impinge on choice. For example, in studying the computational choice of children in the marketplace and at school Carraher, Carraher and Schliemann (1985) found that the choice of computational method was influenced by setting. To suggest that computational choice lies between three alternatives is too simplistic. Ruthven (1998) who studied the use of mental, written and calculator strategies by upper primary students noted that ". . . a refinement of the common-sense trichotomy between mental, written and calculator methods was necessary to take better account of different forms and functions of writing within computation (pp. 29-30).

Theoretical Framework

When faced with a mathematical problem, a person must at some point determine whether or not a calculation is required. Given that a calculation is required, the problem solver must then determine whether an exact or only an approximate answer is needed. Often an estimate is sufficient to answer the question. For example, in order to solve the problem, "Do I have enough money to pay for this trolley-load of groceries?", all that is usually required is an estimate. If a problem requires an exact answer then there are several approaches that might be adopted—

- mental computation;
- pencil and paper (written computation);
- a calculator or computer; or
- some combination of these.

There are several influences that impinge on the decision about which calculation method is used. For example, the nature of the numbers may influence how a person plans to perform the calculation. Ruthven (1995) argued that student attitudes and beliefs will also affect decisions about how to tackle a calculation. Those that have concerns about the legitimacy of using a calculator to solve computational problems may well choose to use an alternative method.

The National Council of Teachers of Mathematics (NCTM, 1989) presented the model shown in Figure 1 as a means of describing the process a person goes through when deciding how to tackle a problem which involves a calculation. Trafton (1994) noted some problems with the NCTM model and modified it to emphasise the role of estimation. The NCTM model tends to be linear in nature, suggesting the use of a single method at any one time, whereas in reality a combination of the above three methods may be employed. Also, estimation should be used in conjunction will all three methods.

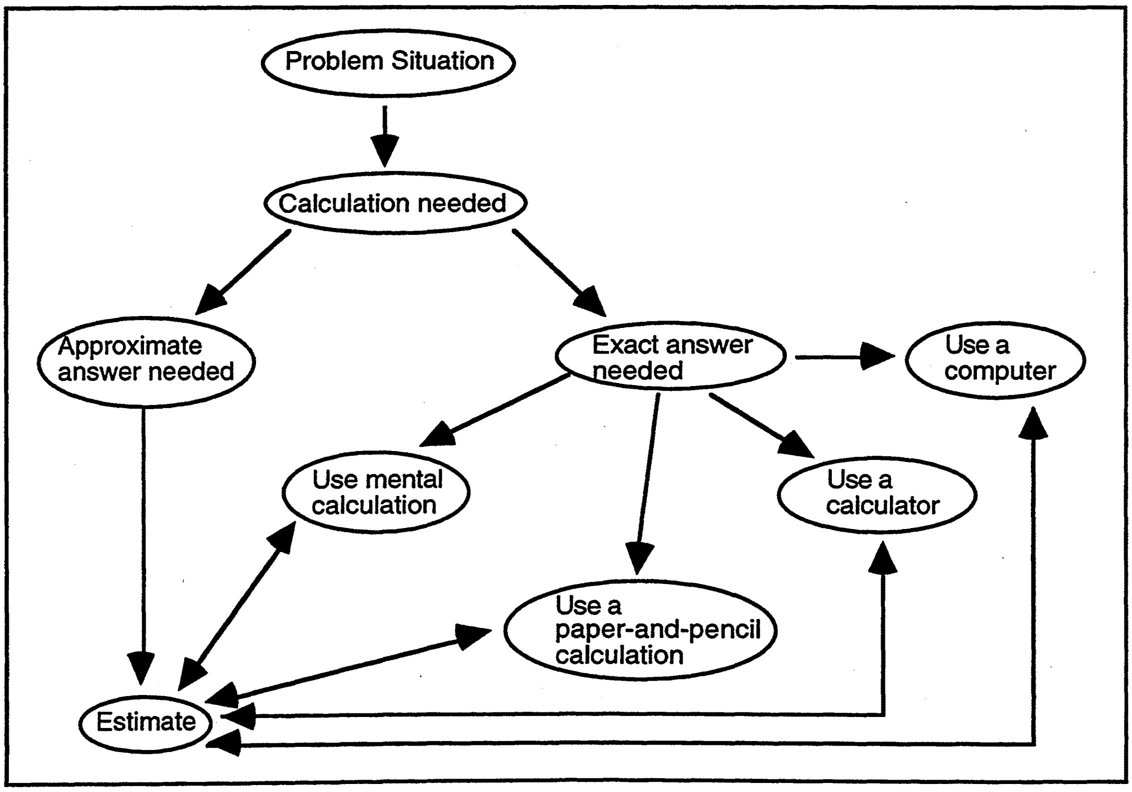

Figure 1. Model to describe computation choice (NCTM 1989, p. 9)

Bobis (1991) linked the use of calculators with number sense suggesting that "students should be taught to check the appropriateness of a computation in three stages — the estimation stage, the calculation stage and the checking stage" (p. 4). Shigematsu, Iwasaki, and Koyama (1994) related mental computation to calculator use with a "metacomputation" framework. They believe that "mental computation will become more than merely a basic skill for paper-and-pencil computation but rather a metacomputation for computation by the calculator" (p. 29). Shumway (1994) expanded on this notion by suggesting that metacomputation might be described as "involving processes and strategies employed to guide computational choices" (p. 194).

Swan and Sparrow (1998) incorporated the work of Bobis (1991) and others in emphasising the ideal of the person involved in a calculation actively monitoring the calculation from any initial estimates right through to checking whether or not the answer is 'sensible'.

> A key aspect of this approach is the involvement of the student throughout all stages of the calculation. While performing the calculation ongoing monitoring should take place. The person carrying out the calculation with the aid of a calculator should not simply push buttons but should monitor each phase of the calculation. (p.166)

In an attempt to better understand the computational process, and in particular the role played by calculators, Bana and Swan (1997) developed a non-linear model designed to take into account some of the complexity of this process. This computation model shown in Figure 2 describes the interplay between the methods of computation. The model also takes into account factors, such as metacognitive strategies, which impinge on calculator use and the computation process generally.

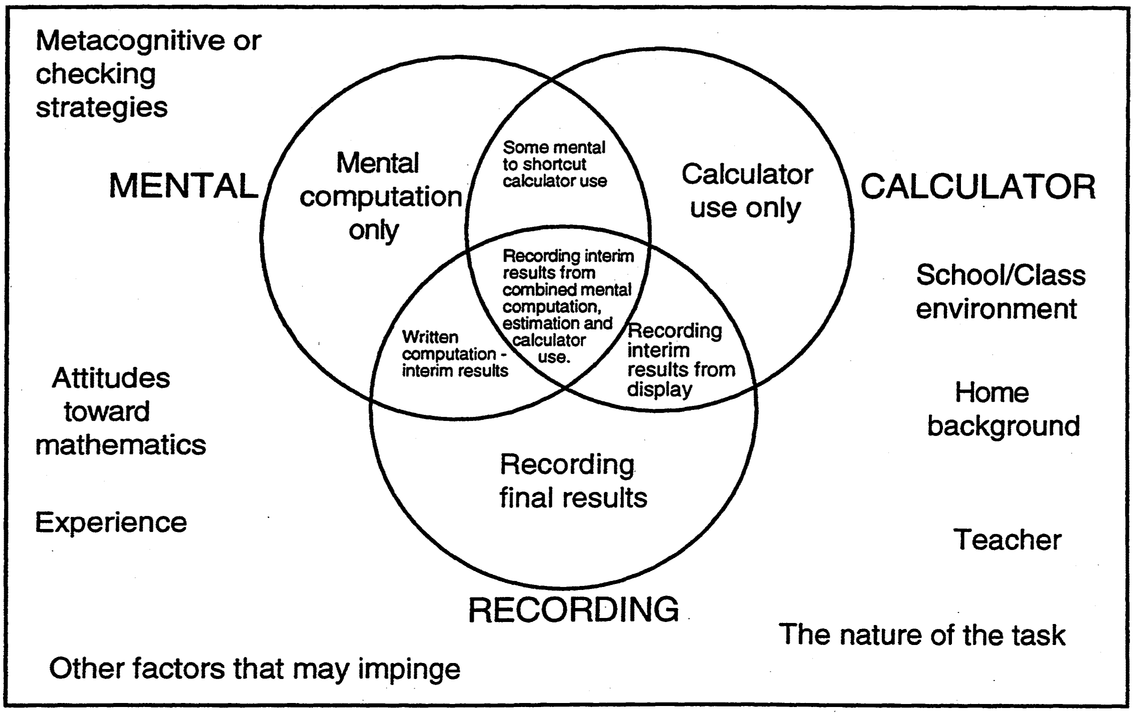

Figure 2: A model of computational processes (Bana & Swan, 1997)

A brief description of the model in Figure 2 follows. The use of the three primary processes of *mental computation*, *calculator use*, and *recording* seems to provide a sound basis for a model of the computation process. The term *mental computation* refers to all work done in the head when performing a calculation. This includes exact mental computations, estimating and checking solutions, and monitoring strategies applied throughout the calculation. *Calculator use* refers to the entering of data and operations into a calculator, the use of calculator memory facilities, the use of a computer in a computation exercise, and the interpretation of displayed results. The term *recording* denotes any informal jottings during the calculation, as well as the recording of more formal algorithmic steps.

The complex nature of computation is such that, apart from mental computation, most computation activity will involve some combination of two or three of the primary processes identified above. For example, written computation is not seen as a unique process, as it generally involves mental computation as well as recording of interim and final results. As more is learned about the computation process the model will be refined to reflect more closely what actually takes place.

The Study

An exploratory study was undertaken to refine an instrument for the main study which will attempt to test the suitability of the theoretical model in Figure 2. A sample of 12 students from Year 5 to Year 7 from two schools with similar profiles were interviewed individually as they attempted a set of 15 computation items. The interviewer and the student sat together at a table that contained pen, paper and a calculator. At the beginning of the interview, students were told that they could solve the problem using whatever method they liked. Once they had attained an answer to the item, the interviewer asked the students why they chose a particular approach to solving the problem and asked them to explain how they went about solving it. Each item was treated in this way. Field notes were taken as the students attempted to solve the problem, and also later when they explained their strategies and processes.

Some General Observations

The preliminary data from our exploratory study are already yielding some very interesting results. From this limited data the following tentative observations have been made:

- Given a free choice of computational method, most students tended to favour mental computation, though this may have been due to the nature of the items. For example, in the item that involved adding 28 and 37, all students chose to solve this problem mentally.
- Students often chose to use the calculator in questions involving "big numbers" and those that involved decimals, or in the words of some students, "because of the dot".
- Items involving simple fractions proved to be very difficult. For example, The children in the sample struggled with the question two-thirds of forty-five. Some indicated that they had little understanding of fractions, having struggled with them in class.
- Some students changed their approach part way through a calculation. This occurred several times, particularly when they got to a point where they realised they had reached their limitations. The item 23·35+34·75 was one example where this happened. Students often tried the calculation mentally, got stuck, and then reached for a calculator.
- Items involving 'carrying' or regrouping were more likely to be completed with pen and paper.
- Several students used a combination of mental methods and informal jottings.
- When students began to run into difficulties or realised they had made a mistake they would often turn to the calculator.

- Some students made use of the calculator only as a checking device.
- All cases appeared to fit the theoretical model of computation processes illustrated in Figure 2.

Data from Selected Items

The following are examples of observations from some of the 15 items used in the exploratory study.

Item — 23·35 + 34·75: One student started to answer this question mentally, realised it involved 'carrying', then chose to use a calculator to complete the question. When probed a little further the student was unsure whether or not the answer produced with the aid of a calculator was correct. He commented that he wasn't "too sure on my points". Another student used the calculator to correctly answer the question and stated that she "hated decimals". Another student who commented that she was "not that good at decimals" used the calculator, obtained the correct result and commented, "that looks about right", after performing the mental calculation, 23 + 34.

Item — 7·41 – 2·5: This item caused some concern for students. One student said she couldn't do it because there were two digits behind the decimal in the first part and only one in the second part. She then went on to use the calculator and found the correct answer.

Item — 1·5 x 20: None of the students used pen and paper methods for this item. One child who used a calculator and got an answer of 30 was surprised indicating that he had expected to get 21. This tends to indicate that he had thought about a possible result prior to the answer appearing on the display. Students using mental methods instantly recognised that 0·5 was one-half, knew that half of twenty was ten, and added ten to twenty to make thirty.

Item — Half of 5 times 2: One student initially chose to use a calculator, then tried a combination of mental computation and calculator but still could not complete the calculation.

Item — 3000 x 4000: Many students chose to complete this problem with the aid of a calculator, which was somewhat surprising given the relatively simple nature of the problem. All those using a calculator explained it was because the numbers were big. Students who chose to use mental computation methods invariably took off the zeros then added zeros. This method did not always produce a correct answer, and when probed it was clear that students often seemed to apply this rule with little or no understanding.

Item — 14 x 9 ÷ 6: This item elicited a number of interesting responses. For example, one student used a formal written algorithm to calculate 14 x 9 and then completed 156 ÷ 6 on the calculator. The student said that the second part was done on a calculator because it involved division. After completing the problem this way the student then used the calculator to work out 14 x 9 and found it to be 126 rather than 156, and then completed the calculation with the aid of a calculator. When probed as to why she had chosen to repeat the calculation her response was vague and didn't indicate any checking procedures such as preliminary estimates that had caused her to redo the calculation. One student used a combination of all three methods. Firstly she mentally determined that 12 x 9 was 108 and then worked out 2 x 9 and added the two results to arrive at the answer of 126. The rest of the calculation, 126 ÷ 6, was completed using pen and paper. Finally the answer was checked using a calculator. A similar technique was used by another student. After completing 14 x 9 mentally the student attempted to complete 126 ÷ 6 using a formal written algorithm, but experienced difficulties and then reached for the calculator to complete the problem. This item certainly caused students to rethink their method of computation. Some realised they were in difficulty or had made a mistake but had problems explaining how they knew they had made a mistake.

Item — 1 x 2 x 3 x 4 x 5 x 6 x 7 x 0: It appears that students often do not take the time to look at a question before making a choice of computational process. Several students initially reached for the calculator, got part way through the solution and abandoned the calculator in favour of mental computation. One student who started the calculation using paper and pencil methods became totally lost as the numbers became larger and gave up before completing the problem. She then opted to use a calculator and discovered that the answer was zero and said, "that's different".

Items — 99c + $2·39; $6·99 + $2·39 + $1·79: These two in-context items provided some interesting data. Students were given some advertisements for chocolate, biscuits and soft drink that were cut from the local paper and asked to calculate the total cost if buying certain combinations of items. A block of chocolate was advertised at $2·39, biscuits at 99c, a carton of soft drink at $6·99 and one bottle of soft drink at $1·79. The first question, 99c + $2·39, was mainly answered using a mental computation method that involved rounding the 99c to one dollar, performing the addition and then compensating for the one cent. Similar reasoning was used to complete the second question with the $6·99 being rounded to $7·00, the $2·39 to $2·40 and the $1·79 being rounded to $1·80. One student completed the problem in written form. Interestingly, no-one used a calculator.

Item — 300 ÷ 6: This item invoked an interesting response from one Year 6 student who commented, "on a test I would do it on paper because I know I am not cheating myself. Using a calculator is a bit like cheating because you don't know the answer until it comes up on the screen". It should be noted, however, that this student eventually used a calculator to attain the correct result.

Where to from here?

Several lines of inquiry have resulted from this exploratory study. The authors are currently gathering further data to evaluate the model postulated above. Of particular interest are the various mixes and blends of mental, written and calculator methods of computation. The aspect of switching from one form of calculation to another is a further avenue to be investigated. The instruments used during the pilot have been refined as appropriate and data collection is under way in the main study. A number of questions are being addressed, and these are listed below.

What is it about a particular question that triggers a child to reach for a calculator? There is some suggestion that large numbers, decimals and fractions may be triggers to pick up a calculator, but clearly there is a difference between an item such as 3000 x 4000 and another like 2387 x 9473, even though we may classify both as four-digit by four-digit multiplication.

What monitoring strategies, if any, do students apply when performing a computation? It appears that, while students may not apply explicit monitoring strategies, there are certain techniques that some students use to check their calculations.

Do computational 'tricks' such as 'take off the zeros' help children or confuse them? McIntosh, De Nardi and Swan (1994) have already noted that "teaching rules (for example 'removing zeros') is as dangerous and self-defeating for mental computation as it is for written computation" (p. 4). Clearly this was the case with the pilot group. Unfortunately onlookers who observe a child using a calculator to complete a computation such as 3000 x 4000 look on with disdain and make the claim that the child is becoming reliant on the calculator to do simple problems.

Do students who have been exposed to a calculator-rich environment tackle problems in a different way from students in calculator-impoverished environments? Indications from the CAN project (Shuard, 1992) are that students have been influenced considerably by their exposure to a calculator-rich environment. At the very least, such students have a good knowledge of how to use a calculator and the limitations of the calculator. Students in this study did not choose to use a calculator to tackle a problem such as $^{2}/_{3}$ of 45. This may have been the case because they did not realise that you could multiply 45 by 2 and then divide by three to obtain an answer.

Does the availability of a calculator influence the way students might think about tackling a problem? Fielker (1992) and Baggett and Ehrenfeucht (1994) refer to the notion of "calculator algorithms" or plans used to solve problems with the aid of a calculator. Often these plans are expressed as keystroke sequences. Fielker has coined the term "calgorithm" to describe this idea. No students in the present study displayed the type of behaviour discussed by Fielker and by Bagget and Ehrenfeucht, but some children in the study chose to use mental computation except when it came to division, in which case they ignored written computation in favour of the calculator.

References

Baggett, P. & Ehrenfeucht, A. (1994). Using calculators in K-5 mathematics: Some classroom observations. (Personal communication)

Bana, J. & Swan, P. (1997). The role of calculators in students' computation strategies. Unpublished paper. Mathematics, Science and Technology Education Centre (MASTEC), Edith Cowan University, Perth.

Bobis, J. F. (1991). Using a calculator to develop number sense. *Arithmetic Teacher, 38*(5), 42-45.

Carraher, T.N., Carraher, D.W. & Schliemann, A.D. (1985). Mathematics in the streets and in the schools. *British Journal of Developmental Psychology, 3,* 21-29.

Fielker, D. (1992). Calculator algorithms. *Micromath, 8*(1), 33-35.

Jones, S. & Tanner, H. (1997). Do calculators count? *Micromath, 13* (3), 31-36.

McIntosh, A., De Nardi, E & Swan, P. (1994) *Think Mathematically!* Melbourne: Longman Cheshire.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Porter, A. (1989). Curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*(5), 9-15.

Ruthven, K. (1998). The use of mental, written and calculator strategies of numerical computation by upper primary pupils within a 'calculator-aware' number curriculum. *British Educational Research Journal, 24*(1), 21-42.

Ruthven, K. (1995). Pressing on: Towards considered calculator use. In L. Burton & B. Jaworski (Eds.), *Technology in mathematics teaching: A bridge between teaching and learning* (pp. 231-256). Bomley, England: Chartwell-Bratt.

Shigematsu, K., Iwasaki, H. & Koyama, M. (1994). Mental computation: Evaluation, curriculum, and instructional issues from the Japanese perspective. In Reys, R. E. & Nohda, N. (Eds) *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States* (pp. 19-30). Reston, VA: NCTM.

Shuard, H., Walsh, A., Goodwin, J. & Worcester, V. (1991). *Calculators, children and mathematics.* Hemel Hempsted, UK: Simon & Schuster.

Shuard, H. (1992). CAN: Calculator use in primary grades in England and Wales. In J.T. Fey & C.R. Hirsch (Eds.), *Calculators in mathematics education: 1992 yearbook* (pp. 33-45). Reston, VA: NCTM.

Shumway, R. J. (1994). Some common directions for future research related to computational alternatives. In Reys, R. E. & Nohda, N. (Eds) *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States* (pp. 187-195). Reston, VA: NCTM.

Sparrow, L. & Swan, P. (1997). *Calculator use in Western Australian primary schools:* Perth: MASTEC.

Swan, P. & Sparrow, L. (1998). Calculators in the Primary Classroom: Brain Rotters or Mind Openers? In N. Ellerton (Ed) *Issues in Mathematics Education: A Contemporary Perspective.* Perth: MASTEC.

Trafton, P. (1994). Computational estimation: Curriculum development efforts and instructional issues. In Reys, R. E. & Nohda, N. (Eds), *Computational alternatives for the twenty-first century: Cross-cultural perspectives from Japan and the United States* (pp. 76-86). Reston, VA: NCTM.

Warren, V. & Ling, J. (1995). Calculators update. *Micromath, 11*(2), 22-24.